# Exercise 22

Some of the highest tides in the world occur in the Bay of Fundy on the Atlantic Coast of Canada. At Hopewell Cape the water depth at low tide is about 2.0 m and at high tide it is about 12.0 m. The natural period of oscillation is a little more than 12 hours and on June 30, 2009, high tide occurred at 6:45 AM. This helps explain the following model for the water depth $D$ (in meters) as a function of the time $t$ (in hours after midnight) on that day:

$$D(t) = 7 + 5\cos[0.503(t - 6.75)]$$

How fast was the tide rising (or falling) at the following times?

(a) 3:00 AM (a) 6:00 AM

(c) 9:00 AM (d) Noon

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**Solution**

Take the derivative of $D(t)$ to get the rate that the depth increases with respect to time.

$$\begin{aligned}
\frac{dD}{dt} &= \frac{d}{dt}\{7 + 5\cos[0.503(t - 6.75)]\} \\
&= 5\frac{d}{dt}\{\cos[0.503(t - 6.75)]\} \\
&= 5\{-\sin[0.503(t - 6.75)]\} \cdot \frac{d}{dt}[0.503(t - 6.75)] \\
&= 5\{-\sin[0.503(t - 6.75)]\} \cdot (0.503) \\
&= -2.515\sin[0.503(t - 6.75)]
\end{aligned}$$

For 3:00 AM, plug in $t = 3$.

$$D'(3) = -2.515\sin[0.503(3 - 6.75)] \approx 2.3909\ \frac{\text{meters}}{\text{hour}}\ \text{(rising tide)}$$

For 6:00 AM, plug in $t = 6$.

$$D'(6) = -2.515\sin[0.503(6 - 6.75)] \approx 0.926439\ \frac{\text{meters}}{\text{hour}}\ \text{(rising tide)}$$

For 9:00 AM, plug in $t = 9$.

$$D'(9) = -2.515\sin[0.503(9 - 6.75)] \approx -2.27647\ \frac{\text{meters}}{\text{hour}}\ \text{(falling tide)}$$

For noon, plug in $t = 12$.

$$D'(12) = -2.515\sin[0.503(12 - 6.75)] \approx -1.20761\ \frac{\text{meters}}{\text{hour}}\ \text{(falling tide)}$$